

Cosmic Strings and Weak Gravitational Lensing

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We study the deflection of light in the background of a “wiggly” cosmic string, and investigate whether it is possible to detect cosmic strings by means of weak gravitational lensing. For straight strings without small-scale structure there are no signals. In the case of strings with small-scale structure leading to a local gravitational attractive force towards the string, there is a small signal, namely a preferential elliptical distortion of the shape of background galaxies in the direction corresponding to the projection of the string onto the sky. The signal can be statistically distinguished from the signal produced by a linear distribution of black holes by employing an ellipticity axis distribution statistic.

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I. INTRODUCTION

There has been renewed interest in cosmic strings [1] as a contributing factor to cosmological structure formation. This resurgence of interest is due firstly to the realization [2] that in many supersymmetric particle physics models, cosmic strings are formed after inflation, and thus contribute to but not completely replace inflationary perturbations as the seeds for structure formation. Secondly, it has recently been realized that models with cosmic superstrings [3] may well be viable [4]. They could, for example, be generated as the remnant of brane annihilation processes in brane inflation models [5], or they may play an important role in inflationary models in warped backgrounds [6]. Cosmic superstrings may also be left behind after the initial Hagedorn phase in string gas cosmology [7], where they would add an additional component to the spectrum of fluctuations produced by thermal string gas fluctuations [8].

Unlike in the original cosmic string models of structure formation (see e.g. [9] and [10] for reviews), where it was assumed that the strings were the sole source of structure formation, in the context of the current models in which cosmic strings arise at the end of inflation, the strings contribute only a small fraction of the total power to the density fluctuations. The most stringent constraints on the fraction of the power which cosmic strings can contribute come from measurements of the angular power spectrum of coordinates cosmic microwave background (CMB) anisotropies. A large fraction f of the power being due to strings is inconsistent with the observed acoustic oscillations in the angular power spectrum. The best current limits on f are [11] $f < 10^{-1}$. Recent work [12, 13] points to the possibility that statistical analyses searching for the line discontinuities [14] in the microwave temperature maps produced by cosmic strings might lead to even tighter limits.

Given their interest from the point of view of particle physics and superstring theory, it is of great interest to develop statistics to search for strings in observational data. There has been a substantial amount of work on

identifying distinctive signals for strings in CMB temperature maps. In this paper we wish to take first steps at exploring another avenue - weak gravitational lensing. Weak gravitational lensing (see e.g. [15] for a general review of the applications of gravitational lensing to cosmology) is emerging as a powerful tool in observational cosmology to search for the distribution of matter in the universe. One of the advantages of weak gravitational lensing over, for example, galaxy redshift surveys, is that light deflection depends on the total mass, not just the luminous mass. As a consequence, cosmic strings (which are also dark in the sense of not emitting light) also lead to gravitational lensing. In fact, due to conical structure of the metric of space-time in the plane perpendicular to a long string [16], the specific lensing pattern produced by a string can lead to interesting strong lensing patterns, e.g. double images (see [17, 18] for claims to have detected such events, claims which were subsequently shown to be incorrect [19]). In this short paper, we take a first step at searching for weak lensing signals from strings.

Cosmic strings [1] are one-dimensional topological defects which arise during phase transitions in the very early universe. Since they carry energy, they will lead to density fluctuations and CMB anisotropies. Causality implies that the network of strings which forms during the phase transition contains infinite strings. Once formed in the early universe, the network of strings will approach a “scaling solution” which is characterized by of the order one infinite string segment in each Hubble volume, and a distribution of cosmic string loops which are the remnants of the previous evolution. In particular, this implies that in any theory which admits cosmic strings, a network of strings will be present at the current time. The network will consist of a small number of strings crossing out entire Hubble volume (these are commonly called the “infinite” strings). The mean curvature radius R_c of these strings will be comparable to the Hubble radius. Thus, for observations on smaller angular and distance scales, these strings can be approximated as straight. In addition, there is a distribution of string loops with radii R between R_c and a lower scale

set by the strength of the gravitational radiation which the loops emit [9]:

$$G\mu R_c \leq R \leq R_c. \quad (1)$$

String loops with smaller radius live for less than one Hubble expansion time. In the above, μ is the mass per unit length of the string and G is Newton's gravitational constant

The distinctive signatures of strings in observational data are a consequence of the specific form of the metric produced by a cosmic string: space perpendicular to a cosmic string is a cone with deficit angle given by [16]

$$\alpha = 8\pi G\mu. \quad (2)$$

In this paper we study the specific signature of this cosmic string metric on weak gravitational lensing. We consider a cosmic string lens and investigate the shape distortions this lens induces for a screen of background galaxies as source objects.

To our knowledge, this is the first investigation of weak gravitational lensing from cosmic strings. Strong lensing signatures (lines of double images) have been studied extensively [20, 21, 22, 23, 24, 25, 26] theoretically, and looked for in observational data sets [27, 28].

The outline of this paper is as follows: we first derive the deflection angle induced by a cosmic string with small scale structure. We then describe a gravitational lensing simulation based on this result, and propose a statistic which could be used to search for this type of effect in future weak lensing surveys.

II. COSMIC STRING LENSING

The deflection of light from distant sources by interposing masses is well understood in the context of General Relativity (GR). There are a variety of lensing phenomena, beginning with strong lensing effects, in which case multiple images of a given source are observed. Other phenomena are microlensing, an increase in the luminosity of a source as a lensing mass passes close to the line of sight between observer and source, and weak lensing which is the shape distortion in the images of an extended source. Since all mass and not only visible matter lead to lensing, weak gravitational lensing [29] offers the hope of providing a tool for mapping out the distribution of dark matter in the universe, amongst other applications. Here, we will study weak lensing imprints of cosmic strings, a specific class of dark objects.

In the usual theory of weak gravitational lensing [29], lenses are characterized by their shear $\bar{\gamma}$ and their magnification μ . These correspond, roughly speaking, to the distortion and magnification produced in the image. Since weak lensing effects are small and the source distribution is a priori unknown, searching for weak lensing relies on a statistical approach to discriminate between

weakly lensed and un-lensed regions of the sky. The success of a weak lensing survey is therefore dependent on the statistic chosen to analyze the data.

In the following, we imagine a screen of background galaxies, modelled as extended ellipsoidal objects. The light from these galaxies is lensed by a foreground cosmic string.

Let us first consider the metric of a straight string without small-scale structure (no “wiggles”) which is characterized by a string tension T which is equal in magnitude to the mass per unit length μ . In cylindrical coordinates, the metric of a such a straight string which we take to be oriented along the z-axis is

$$ds^2 = -dt^2 + dr^2 + dz^2 + (1 - 8G\mu)r^2 d\theta^2, \quad (3)$$

where G is the gravitational constant and μ is the mass per unit length of the string [16]. This is nothing but the Minkowski metric with a deficit angle $\alpha = 8\pi G\mu$. Since the metric is locally flat, no shape distortion is induced by such a string.

Thus, we go on and consider a wiggly cosmic string (tension T smaller than the mass per unit length μ) lying perpendicular to the optical axis, and derive the consequences for weak gravitational lensing in this space-time. We will use standard notation where D_S is the distance from observer to source plane and D_L the distance from observer to lens. For convenience, in cylindrical coordinates the z axis will lie along the direction of the string. The optical axis is chosen as the x direction. Consequently, the source and image planes will be parallel to the y-z plane, consistent with this choice of coordinates.

The deflection angle is defined as in the Schwarzschild case, namely as the deviation of a null geodesic from a source at infinity to an observer at infinity but propagating in the wiggly string metric. The deflection due to the small scale structure of the string is then defined as the deviation from null geodesics propagating in a straight string space-time.

A. Null Geodesics

For an infinite string lying along the z-axis the linearized wiggly string metric in cylindrical coordinates is

$$ds^2 = -(1 + h_{00})dt^2 + dr^2 + (1 - h_{00})dz^2 + \omega^2 r^2 d\theta^2, \quad (4)$$

where

$$h_{00} = 4G(\mu - T)\ln(r/r_0), \quad (5)$$

$$\omega^2 \equiv 1 - 4G(\mu + T), \quad (6)$$

G is the gravitational constant, and r_0 an integration constant [16] which will not be relevant in our analysis.

Using the Lagrangian

$$\mathcal{L} = \frac{1}{2}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu \quad (7)$$

and the corresponding Euler-Lagrange equations we find the constants of motion

$$L \equiv \omega^2 r^2 \dot{\theta}, \quad (8)$$

$$K \equiv (1 + h_{00})\dot{t}, \quad (9)$$

$$M \equiv (1 - h_{00})\dot{z}. \quad (10)$$

Making use of (8), (9) and (10) we can find an effective Lagrangian

$$\mathcal{L}_{Eff} = \frac{1}{2} \left(-\frac{K^2}{(1 + h_{00})} + \dot{r}^2 + \frac{M^2}{(1 - h_{00})} + \frac{L^2}{\omega^2 r^2} \right). \quad (11)$$

Using the condition $2\mathcal{L}_{Eff} = 0$ for null geodesics, we obtain the equation of motion in terms of the radial variable,

$$\dot{r}^2 = \frac{K^2}{(1 + h_{00})} - \frac{M^2}{(1 - h_{00})} - \frac{L^2}{\omega^2 r^2}. \quad (12)$$

We now reparametrize in terms of θ , defining $r' \equiv \dot{r}/\dot{\theta}$. Using the change of variables $r \rightarrow 1/u$ and differentiating once, (11) can be rewritten

$$u'' = -\omega^2 u - \frac{2G(\mu - T)\omega^4}{L^2 u} \left[\frac{K^2}{(1 + h'_{00})^2} + \frac{M^2}{(1 - h'_{00})^2} \right], \quad (13)$$

where $h'_{00} = 4G(\mu - T)\ln(u_0/u)$.

B. Zeroth Order Solution (Straight String Contribution)

Now proceed perturbatively by setting

$$u \equiv u_{(0)} + u_{(1)}, \quad (14)$$

where the subscript denotes the power in our expansion parameter, namely the magnitude of the small scale structure, $G(\mu - T)$.

At zeroth order, (13) reduces to

$$u''_{(0)} = -\omega^2 u_{(0)}. \quad (15)$$

The initial conditions are specified by demanding $u(0) \equiv b^{-1}$, where b is the impact parameter. Solving (15) yields

$$u_{(0)}(\theta) = b^{-1} \cos(\omega\theta). \quad (16)$$

C. First Order Solution (Contribution of Wiggles)

Expanding (13) to first order and substituting the zeroth order result (16), we find

$$u''_{(1)} = -\omega^2 u_{(1)} + \frac{C}{\cos(\omega\theta)}, \quad (17)$$

where

$$C \equiv -\frac{2G(\mu - T)\omega^4 b(K^2 + M^2)}{L^2}. \quad (18)$$

This has the general solution

$$u_{(1)}(\theta) = C_1 \cos(\omega\theta) + C_2 \sin(\omega\theta) + \frac{C}{\omega^2} (\omega\theta \sin(\omega\theta) + \cos(\omega\theta) \ln(\cos(\omega\theta))), \quad (19)$$

where C_1, C_2 are integration constants. Applying the boundary conditions

$$u_{(1)}(0) = u'_{(1)}(0) = 0 \quad (20)$$

at the point of nearest approach we find

$$u_{(1)}(\theta) = \frac{C}{\omega^2} (\omega\theta \sin(\omega\theta) + \cos(\omega\theta) \ln(\cos(\omega\theta))). \quad (21)$$

D. Deflection Angle

To calculate the deflection angle we consider the deviation

$$\Delta\theta \equiv \frac{\pi}{2} - \theta \quad (22)$$

at small values of u ($r \rightarrow \infty$). This quantity is then doubled since the deflection angle is defined as the deviation due to light propagating from $-\infty \rightarrow \infty$.

First, consider the deflection angle for a straight string without small-scale structure, i.e. working with the unperturbed solution $u_{(0)}$. Setting $u_{(0)} = 0$ and expanding by using the fact that $\Delta\theta \ll 1$ we find a deflection angle

$$\alpha_s = 2\Delta\theta = 2\pi G(\mu + T), \quad (23)$$

the standard result.

Next, we move on to the case of the wiggly string. As above, we set $u = u_{(0)} + u_{(1)} = 0$ for $\theta \approx \frac{\pi}{2}$ and solve for $\Delta\theta$ (defined above). In this approximation $\sin(\omega\theta) \approx 1$ and $\cos(\omega\theta) \approx \omega\Delta\theta$. Hence $u_{(0)} = b^{-1}\omega\Delta\theta$ and the leading term for the first order perturbation is

$$u_{(1)} \approx \frac{C\pi}{2\omega}. \quad (24)$$

This yields a deviation

$$\Delta\theta = \frac{-Cb\pi}{2\omega^2}, \quad (25)$$

and a deflection angle

$$\alpha_w = 2\Delta\theta \approx -2G(\mu - T)\omega^2 b^2 \pi \left(\frac{K^2 + M^2}{L^2} \right). \quad (26)$$

Using the null condition and evaluating the constants of the motion at $\theta = 0$ yields the deflection angle due to string structure

$$\alpha_w = \frac{2\pi G(\mu - T)}{\omega^2} \left(2 \left(\frac{\dot{z}}{b\dot{\theta}} \right)^2 + 1 \right). \quad (27)$$

This is a constant deflection term, plus a term that is position dependent which may lead to some shearing.

E. Initial Conditions

We evaluate the initial conditions for the zeroth order geodesic, and use these to calculate the first order deflection. At zeroth order, geodesics are straight lines so we will work in Cartesian coordinates. Parametrizing the geodesic for $0 < \lambda < 1$, where $\lambda = 0$ corresponds to the source position and $\lambda = 1$ corresponds to the observer yields

$$x = (D_L - D_S) + \lambda D_S, \quad (28)$$

$$y = y_0 - \lambda y_0, \quad (29)$$

$$z = z_0 - \lambda z_0. \quad (30)$$

From the geometry (Fig. 1) we find

$$\theta = \Psi - \tan^{-1} \left(\frac{y}{x} \right), \quad (31)$$

where $\Psi(b, D_L)$. Differentiation with respect to λ yields

$$\dot{\theta} = \frac{y\dot{x} - \dot{y}x}{x^2 + y^2}. \quad (32)$$

Substituting (30) and evaluating at $\theta = 0$ yields

$$\dot{\theta} = \frac{D_S^2 + y_0^2}{D_L y_0}. \quad (33)$$

Substituting into (27) yields

$$\alpha_w = \frac{2\pi G(\mu - T)}{\omega^2} \left(\frac{2z_0^2}{(D_S^2 + y_0^2)} + 1 \right). \quad (34)$$

III. RESULTS

A. Ellipticity Axis Distribution as a Weak Lensing Statistic

In order to search for cosmic string lensing we propose using the ellipticity axis distribution (EAD) defined as follows. Consider an elliptical object in the image plane with ellipticity $\epsilon \equiv b/a$ where b, a are the semi-minor and semi-major axes respectively. Define the ellipticity vector $\vec{\epsilon}$ as a unit vector lying in direction of the major axis of the ellipse. Now define the ellipticity axis angle (EAA) ϕ via

$$\phi = \cos^{-1}(\vec{\epsilon} \cdot \vec{z}), \quad (35)$$

where we have chosen to measure the angle with respect to the unit vector \vec{z} . The ellipticity axis angle is hence a measure of the orientation of an ellipse's major axis in the image plane. We will call the distribution of EAA for a set of objects the ellipticity axis distribution (EAD).

In an un-lensed region, one would expect a uniform EAD due to isotropy. However, we will show that cosmic string lensing is predicted to produce a peak in the EAD for a subset of sources.

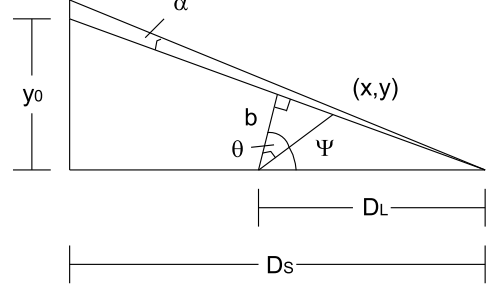


FIG. 1: The geometry of the problem, where the azimuthal angle $\theta = 0$ at the point of closest approach.

B. Shearing of a nearly circular source

We now examine how (34) leads to a specific signature in the EAD of nearly circular sources. We will call a source nearly circular if it is to first order circular in shape (i.e. $z^2 + y^2 \approx r^2$), but unlike a circle has a well defined EAA. The radius of this source is r .

Consider such a source in Cartesian coordinates centered at $(D_S, 0, D_S)$. Points on the surface of the object have coordinates

$$(D_S, y, D_S + z) = (D_S, r \sin(\phi), D_S + r \cos(\phi)), \quad (36)$$

for $0 < \phi < 2\pi$. We want to find the point where the radius change of the image is maximized. Expanding (34) to first order in r we find the non-constant part of the deflection to be

$$\alpha_\Delta = k r \cos(\phi), \quad (37)$$

where $k = \frac{8\pi G(\mu - T)}{\omega^2 D_S}$. The radius of the image point is then approximately $R^2 = z^2 + (y + \alpha_\Delta D_S)^2$ where we

have used the fact that the z coordinate is unaffected by the lens and the non-constant shift in the y direction is $\Delta y \approx \alpha_\Delta D_S$. Expanding to first order in α_Δ yields

$$R^2 = r^2 + 2kr^2 D_S \sin(\phi) \cos(\phi), \quad (38)$$

where we have used the nearly circular property of the source. Differentiating we find

$$R \frac{dR}{d\phi} = k(\cos^2(\phi) - \sin^2(\phi))r^2. \quad (39)$$

Hence R is extremized at $\phi = \pi/4$, and it is easy to check that this is a maximum. For nearly circular sources this admits the possibility that the EAD will have a sharp peak because the EAA of the images will preferentially be at $\phi = \pi/4$ since this is the angle at which the greatest shearing occurs. In contrast, an object centered at the position $(D_S, 0, -D_S)$ would experience a shear which would lead to an EAD peaked at $\phi = -\pi/4$.

C. Lensing simulation

The goal was to determine the threshold ellipticity of galaxies which would produce a detectable signal in the EAD. We begin with a short description of the code.

The source plane is created by forming a regularly spaced lattice and placing a galaxy centered at each lattice point. The lensing map is applied to each galaxy in the source plane to produce the image plane. The ellipticity angle of each galaxy is measured by identifying the major axis and using (35). These results are compiled to produce the EAD.

We fixed the small parameter responsible for the effect to be $4\pi G(\mu - T) = 10^{-6}$, a value which corresponds to the upper range allowed by the current constraints on μ . The source plane was fixed at a distance of redshift $z = 0.1$, the typical redshift of galaxies in current large-scale galaxy redshift surveys, and the lens plane at a redshift of $z = 0.05$. The mean galaxy size was chosen to be $r = 30000\text{pc}$, similar to the size of our Milky Way galaxy.

Galaxies have an intrinsic ellipticity which will act as background noise to our analysis. We have chosen the ellipticity magnitude ϵ to be randomly chosen in the range $\epsilon \in [t, 1]$ according to a uniformly distributed measure, where $t \approx 1$, i.e. nearly spherical objects, needed to be taken in order to be able to obtain a weak lensing signature from strings. The axis of each source object was randomly chosen in the range $\phi \in [-\pi/2, \pi/2]$, again according to a uniform measure (Fig. 2). The cutoff ellipticity parameter t was increased until a detectable signal was achieved in the EAD of the objects in the image plane. The number of objects in the source plane was taken to be 10,000.

By varying the parameter t , it was found that a detectable signal in the EAD could be measured for $t = 0.995$, for objects at a distance $z \approx D_S$ (Fig. 3). Decreasing the strength of the small scale structure led to

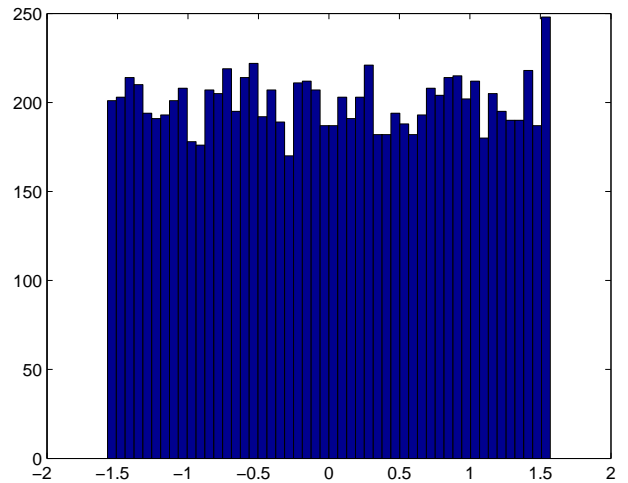


FIG. 2: EAD of the source plane corresponding to a random choice of ellipticity axes in the interval $[-\pi/2, \pi/2]$ (uniform measure).

a corresponding increase in t , the threshold ellipticity. At fixed t , decreasing the value of z caused the signal to gradually vanish.

D. Search Strategy

To search for this effect one might first divide the sky into a grid and find the EAD of sources with ellipticity $\epsilon \in [t, 1]$ in each grid square. Then determine the peak in the EAD of each grid square and search for a linear distribution of grid squares where the EAD peak varies from $-\pi/4$ to $\pi/4$ across the distribution. This would signal the presence of a cosmic string stretching across the linear distribution of grid squares.

Searching for this lensing effect can therefore be used to set bounds on the small scale structure of any potential cosmic strings in our Hubble volume. In the optimistic case that this effect is observed, it would provide a motivation and starting point to search for the unique strong lensing signature of a straight cosmic string.

It is important to point out that a combination of ordinary Schwarzschild lenses could in principle produce a similar EAD signature. Since the lensing mass is not uniformly distributed along the z direction, the lensing angle distribution would be less regular, thus allowing a discrimination between the signals from a string and from a line of Schwarzschild lenses. However, because of the huge degeneracy of this problem, it seems premature to attempt an analysis without actual data. We conclude that this lensing signature be used as a first step in subsequent analyses of weak lensing surveys, and that more rigorous analysis, in particular searching for the strong lensing effect caused by the deficit cone, be used to pin-

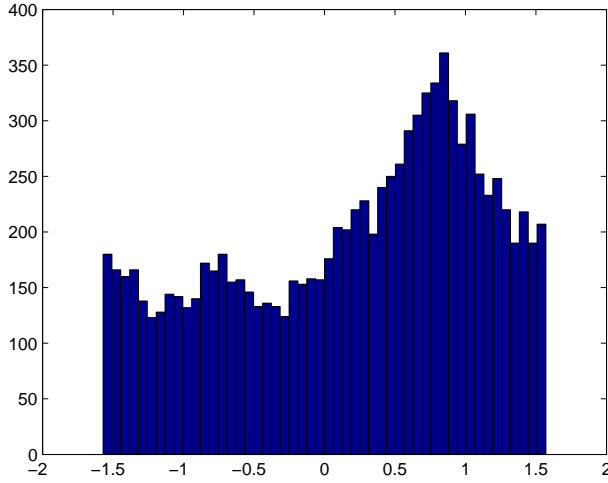


FIG. 3: EAD of the image plane for sources far from the optical axis ($z \approx D_S$) and EAA randomly chosen in the range $[0.995, 1]$ (uniform measure). As predicted, a sharp peak appears at $\phi \approx \pi/4$.

point the exact location of the string.

IV. CONCLUSIONS

In this work we have studied the gravitational lensing by a straight cosmic string containing small-scale struc-

ture which leads to a string tension which is less than the mass per unit length of the string, and thus induces a net gravitational force towards the string which test particles feel.

Next, we studied potential weak lensing signatures of such wiggly strings. We found a shape distortion which is proportional to $G(\mu - T)$ but depends on the location of the source relative to the line of sight between the observer and the string. Only objects which are displaced in direction of the string relative to the line of sight give rise to a shape distortion. The shape distortion increases linearly as the distance in string direction increases.

The specific distribution of the weak lensing distortion in the image plane in principle can be used to provide a signature for cosmic strings. However, in practise the signal appears to be too small to be useful. Even for optimal choices of parameters (galaxy redshifts comparable to those of galaxies in the largest current redshift surveys), distance from the line of sight comparable to the distance of the string from the observer, large amount of small scale structure and string tension close to the current observational bounds, the intrinsic ellipticity of the background galaxies needs to be very close to one (i.e. the shape needs to be very close to spherical) in order for the cosmic string signal to stand out.

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